Synthesis of Optimal Thermal Membrane Distillation Networks

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Thermal membrane distillation (TMD) is an emerging separation method which involves simultaneous heat and mass transfer through a hydrophobic semipermeable membrane. Traditionally, studies of this technology have focused on the performance of individual modules. Because of purity and recovery requirements, multiple TMD modules may be used in various configurations including series, parallel, and combinations. Furthermore, there may be a need to reroute streams from one module to another or to recycle a stream to the same unit. The objective is to develop a systematic approach to synthesize an optimal TMD network. A structural representation is developed to embed potential configurations of interest. A mathematical formulation is developed to transform the design problem into an optimization task that seeks to minimize the cost of the system. Two case studies are presented to illustrate the applicability of the developed approach and its merit over conventional design scenarios. © 2014 American Institute of Chemical Engineers *AIChE J*, 61: 448–463, 2015
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Introduction

Thermal membrane distillation (TMD) is emerging as a promising technology that can achieve high levels of separation. TMD has similarities to both membrane separation and distillation. Separation is based on difference in vapor liquid equilibrium and preferential permeation through a membrane. The driving force is the difference in the partial vapor pressure across a microporous hydrophobic membrane.[1,2] The feed is preheated to a temperature below boiling point. Then, the vapor diffuses through the membrane and is condensed and collected as permeate. TMD offers several benefits over other existing desalination technologies including very high theoretical rejection of ions, macromolecules, colloids, cells, and other nonvolatiles, low level heating and moderate operating temperature and pressure, compact size, ability to handle concentrated feeds and it is relatively simple to increase capacity by adding TMD modules.[3,4] Membrane distillation systems may be classified into four configurations, a brief description can be found in literature.[4,5] Many studies have shown the applicability of the TMD technology to produce ultrapure water and to remove nonvolatile solutes in aqueous solutions such as salts, sugar, fruit juices, and blood.[6–8] Furthermore, TMD has been shown to work well in water-treatment applications such as desalination and water purification.[9–12] Elsayed et al. developed an approach for modeling TMD units and thermal coupling with industrial facilities, which has the potential to decrease the unit cost of desalination from 7.56 to 1.05 \$/m^3.[13]

It is worth noting that although TMD units offer high separation levels, there are several applications where a single stage is not sufficient to reach the desired purity levels. In such cases, permeate staging through layers of TMD units in series may be required. Furthermore, if the desired recovery of permeate is not achieved in one stage, reject (brine or retentate) staging may be required. Also, when brine discharge is limited or when levels approaching zero-liquid discharge are needed, there may be need for reject staging. When the high-concentration reject is fed to a TMD unit, the concentration of permeate may be too high. This, in turn, requires further permeate staging. All of these issues call for the need to design networks of TMD units involving configurations in series and parallel. This issue has not been

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addressed in literature. This work is aimed at synthesizing networks of TMD units.

While there has been no prior work on the synthesis on integrated TMD networks (TMDN), several papers have been published in the area of synthesizing other types of membrane networks. El-Halwagi introduced a technique for synthesizing reverse osmosis networks (RONs) that accounts for network configuration and integration of multiple types of units.[14,15] Srinivas and El-Halwagi addressed the problem of synthesizing pervaporation networks using mixed-integer programming techniques.[16] Crabtree et al. developed an optimization procedure for synthesizing gas-permeation networks.[17] Zhu et al. proposed a multiperiod approach to the design and scheduling of flexible RONs.[18] See et al. presented an optimization model for reverse osmosis (RO) desalination where the effect of the network configuration on the optimal cleaning schedule and total cost of a membrane desalination plant was considered.[19] An optimal design strategy was proposed by Qi and Henson for membrane networks for separating multicomponent gas mixtures.[20] Maskan et al. presented a model using individual RO units to design optimal network configurations.[21] Kookos presented an approach for optimizing the selection of membrane material along with the structure of the membrane network.[22] Marriott and Sorensen introduced a design procedure using genetic algorithms for membrane systems such as pervaporation.[23] Uppaluri et al. proposed a robust stochastic technique using a simulated annealing procedure for minimizing the annualized cost of gas permeation networks.[24] Karuppiah et al. presented an optimization approach for synthesizing water treatment systems accounting for RO units,[25] whereas Abejón et al. considering RO units for ultrapurification of chemicals.[26] Alnouri and Linke proposed a systematic process synthesis and optimization approach that takes into consideration multiple water quality parameters in synthesizing RO desalination networks.[27,28] Du et al. proposes a multiobjective optimization approach for synthesizing RO networks for seawater desalination.[29] Almansoori and Saif presented an optimal integrated RO system and a pressure retarded osmosis for seawater desalination and power generation.[30] Saif et al. presented an approach for the optimal design of RO networks considering variations in the stream properties.[31] Dahdah and Mitsos introduced a methodology to identify improved thermal-based desalination structures, the model considers either a multieffect distillation system or a stage in a multistage flash system to determine the best configuration.[32]

As discussed earlier, several applications require the use of multiple TMD modules that may be arranged in series, parallel, or a combination. In addition to the configuration of the TMD units, the use of heaters and coolers throughout the network is an important design variable. To enhance the permeation flux, the feed to each TMD may be heated to some optimal temperature. The vapor permeate has to be condensed. Therefore, the placement and duties of heaters, coolers, and heat exchangers (integrating heat transfer) must be considered. Furthermore, because of the relatively low water recover per pass of TMD, it may be necessary to increase the feed flow rate to each stage through recycling the reject stream. The extent and allocation of reject recycle are optimization variables. These issues contribute to the complexity of synthesizing optimal TMDN. A TMDN is composed of multiple TMD modules, pumps, heaters, condensers, mixers, and splitters. This article first describes the problem of synthesizing a system of TMD modules and then presents a systematic procedure for designing an optimal TMDN based on a new superstructure and a mathematical programming formulation.

Problem Statement

The synthesis of a TMDN can be stated as follows: Given a feed flow rate Q_F and feed concentration C_F, it is desired to synthesize a TMDN to obtain a permeate stream (e.g., clean water) with targeted flow rate and purity at minimum cost. The TMDN consists of several interconnected TMD modules, pumps, heaters, and condensers, whose structure, interconnection, configuration, sizes, and operating conditions must be optimized. The total annual cost includes the capital costs for the units (fixed and variable costs) as well as the operating cost minus the revenues obtained for the sale of the permeate and/or the concentrated retentate.

The TMDN synthesis problem involves addressing several design challenges. The first issue is the need to consider many configurations of the system. Some common configurations for TMDN are in series, parallel, and combined arrangements. Figures 1 and 2 illustrate examples of potential configurations for TMDN. The series arrangement (see Figure 1a) is the one typically used when the desired extent of separation exceeds the maximum ability of one stage. TMD modules are arranged in parallel (Figure 1b) when the flow rate of the feed to the network exceeds the capacity of an individual module. The tapered arrangement (Figure 2) of TMD units (sometimes referred to as the "Christmas tree" arrangement) is a hybrid of the series and parallel configurations to address the need for permeate recovery and purity as well as constraints on liquid discharge.

In addition to the design challenges associated with the selection of the system configuration, there are several other design decisions that must be included:

- The number of modules to be used.
- The total membrane area for the network.
- The configuration for the modules (e.g., in series, parallel, and combination).
- The optimal values of heating and cooling duties.
- The extent and allocation of reject recycle.
- The optimal values of operating variables for each module (temperature and concentration).

The aforementioned challenges call for the development of a systematic procedure and a mathematical formulation that can address these design tasks and provide optimal solutions to the synthesis of the TMDN problem. This is presented in the next section.

Design Approach

The source-sink approach presented by El-Halwagi et al. was used to represent the network configuration.[33] A source is a process stream that is rich in the constituent(s) that has to be removed in the separation task. For this case, we propose a building block consisting of a TMD module (see Figure 3). The TMD module offers advantages of simplicity of construction, operation, and maintenance as well as consistent performance. In the TMD module, the feed water is preheated. Enough heat should be provided to induce the evaporation at a moderate temperature. The heat to be

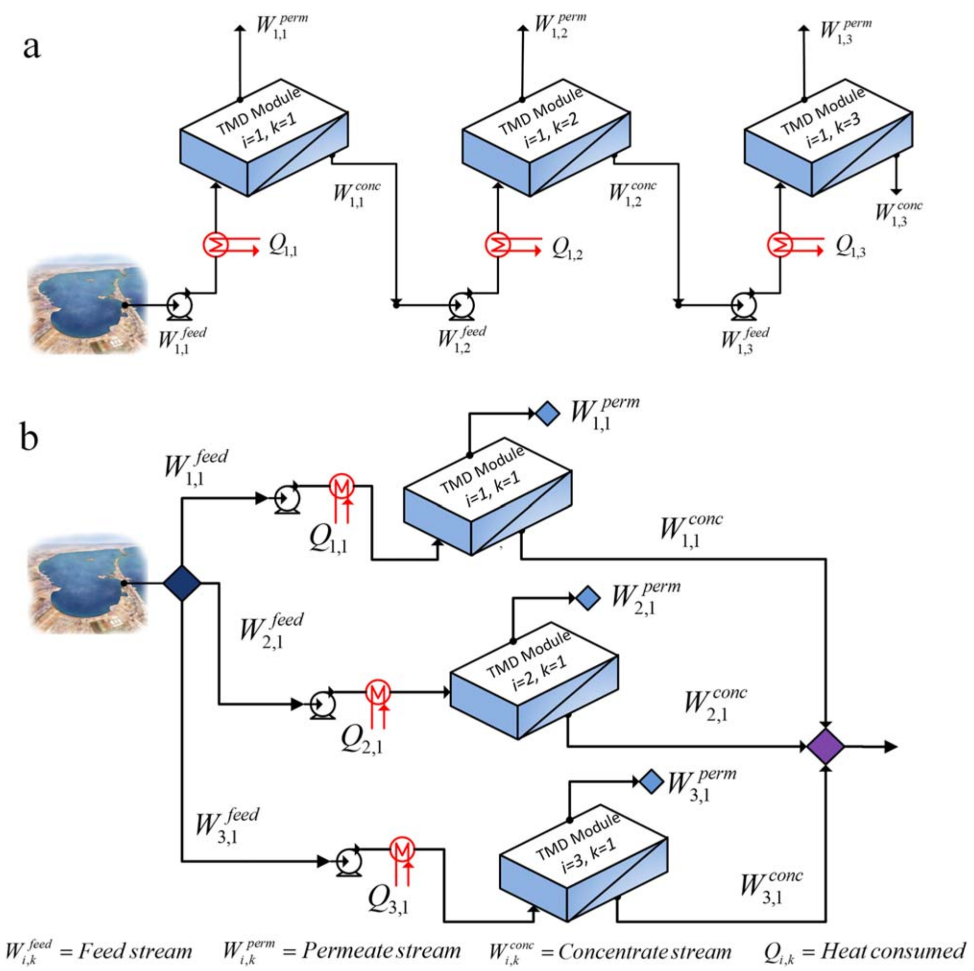

Figure 1. Schematic representation of TMD modules in (a) series arrangement and (b) parallel arrangement.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

supplied and the temperature of the stream fed to the TMD unit are optimization variables. The water vapor travels through the membrane and is condensed on the permeate side using a recirculating permeate-sweeping liquid which is colder than the feed. The size of each element is an optimization variable (including a zero size, which indicates that the element does not exist). Each TMD unit produces two sources (permeate and reject). Either one or both may be rerouted back to the network to be assigned to new sinks.

Figure 4 shows the proposed superstructure for designing an integrated TMDN. The feed may be split into several fractions assigned to the TMD units. In turn, each unit produces two sources (permeate and reject) that may be recycled or fed to another unit. The problem consists of determining the optimum network configuration, unit sizes, stream allocation, and operating conditions. In the proposed formulation, the following assumptions were used:

- The separation performance of the TMD modules is a function of temperature. This is due to the fact that the flux of permeate is a function of temperature.
- The temperature at a module is independent of that at other modules.
- The specific heat of the mixture does not vary with the temperature.
- Only a single component is considered in the total feed concentration.

Optimization Model for TMDN

This section presents the proposed optimization model for designing the TMDN based on the superstructure shown in Figure 4. The indexes used in this model are the following: i represents the number of TMD units connected in parallel arrangement, k indicates the number of units connected in series, and i' is used to indicate a subsequent line of TMD modules. For better understanding of the model, the super indexes are also defined: feed is the raw feed stream from initial or previous stage, TMD represents the streams that feed the TMD unit, conc is the stream that leaves the TMD unit before the recycle divisor, recycle is a recycled process stream, rej is the rejected stream from TMD process which is sent to another stage, and perm is the permeate stream.

Mass balance, salt balance, and energy balance in the recycle mixers

A feed stream (e.g., seawater) is transported directly from the origin (e.g., sea) to the storage units. The main process starts with a stream mixer for which the mass balance in this unit is the sum between the feed flow rate ($w_{i,k}^{\text{feed}}$) plus the recycle stream ($w_{i,k}^{\text{recycle}}$) to give the feed flow rate to the TMD unit ($w_{i,k}^{\text{TMD}}$)

$$w_{i,k}^{\text{TMD}} = w_{i,k}^{\text{feed}} + w_{i,k}^{\text{recycle}}, \forall i \in I, \forall k \in K \tag{1}$$

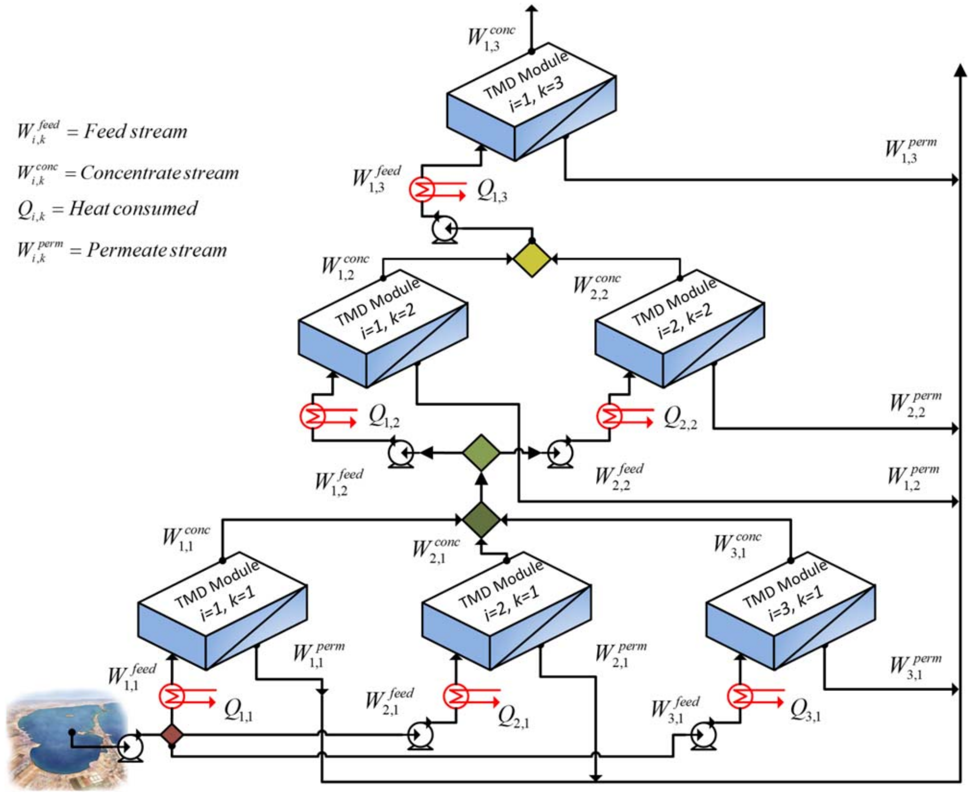

Figure 2. Tapered TMDN configuration.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Then, the solute balance for the recycle mixer is given as follows

$$w_{i,k}^{\mathrm{TMD}} z_{i,k}^{\mathrm{TMD}} = w_{i,k}^{\mathrm{feed}} z_{i,k}^{\mathrm{feed}} + w_{i,k}^{\mathrm{recycle}} z_{i,k}^{\mathrm{rej}}, \forall i \in I, \forall k \in K \quad (2)$$

where $z_{i,k}^{\mathrm{TMD}}$ is the concentration of the stream ($w_{i,k}^{\mathrm{TMD}}$) to be fed into the TMD unit, $z_{i,k}^{\mathrm{feed}}$ is the concentration of the raw feed ($w_{i,k}^{\mathrm{feed}}$) and $z_{i,k}^{\mathrm{rej}}$ is the concentration in the recycled stream from the concentrate ($w_{i,k}^{\mathrm{recycle}}$).

The energy balance in the mixer can be stated as follows

$$w_{i,k}^{\mathrm{TMD}} \mathrm{Cp}_{i,k}^{\mathrm{TMD}} (t_{i,k}^{\mathrm{mix}}) = w_{i,k}^{\mathrm{feed}} \mathrm{Cp}_{i,k}^{\mathrm{feed}} (t_{i,k}^{\mathrm{in}}) + w_{i,k}^{\mathrm{recycle}} \mathrm{Cp}_{i,k}^{\mathrm{rej}} (t_{i,k}^{\mathrm{rej}}), \quad \forall i \in I, \forall k \in K \quad (3)$$

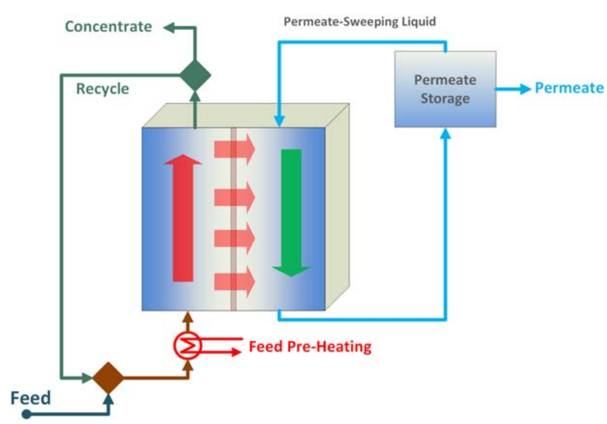

Figure 3. TMD scheme with reject recycle and permeate sweeping liquid.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

where the temperature $t_{i,k}^{\mathrm{mix}}$ is the result of the temperatures from the mixed streams $w_{i,k}^{\mathrm{feed}}$ and $w_{i,k}^{\mathrm{recycle}}$, and $t_{i,k}^{\mathrm{in}}$ and $t_{i,k}^{\mathrm{rej}}$ are the temperatures of the raw feed and the recycled stream, respectively.

Energy balance in the heaters

The mixed stream is preheated using heating utilities, the energy balance in the heater can be described with the following equation

$$w_{i,k}^{\mathrm{TMD}} \mathrm{Cp}_{i,k}^{\mathrm{TMD}} (t_{i,k}^{\mathrm{TMD}} - t_{i,k}^{\mathrm{mix}}) = Q_{i,k}^{\mathrm{Heating}}, \forall i \in I, \forall k \in K \quad (4)$$

The total water flow rate ($w_{i,k}^{\mathrm{TMD}}$) is heated from a temperature $t_{i,k}^{\mathrm{mix}}$ to the operating temperature in the TMD unit ($t_{i,k}^{\mathrm{TMD}}$), where $Q_{i,k}^{\mathrm{Heating}}$ is the heat required to reach the TMD operating temperature.

Mass balance and salt balance for TMD units

For the process, the amount of feed assigned to the TMD unit ($w_{i,k}^{\mathrm{TMD}}$) is equal to the permeate ($w_{i,k}^{\mathrm{perm}}$) plus the rejected stream ($w_{i,k}^{\mathrm{rej}}$)

$$w_{i,k}^{\mathrm{TMD}} = w_{i,k}^{\mathrm{perm}} + w_{i,k}^{\mathrm{rej}}, \forall i \in I, \forall k \in K \quad (5)$$

In the case of desalination, the salinity in seawater or brackish water depends on the region and affects the amount of the obtained permeate at the end of the treatment stages ($w_{i,k}^{\mathrm{perm}}$)

$$z_{i,k}^{\mathrm{TMD}} w_{i,k}^{\mathrm{TMD}} = z_{i,k}^{\mathrm{perm}} w_{i,k}^{\mathrm{perm}} + z_{i,k}^{\mathrm{rej}} w_{i,k}^{\mathrm{rej}}, \forall i \in I, \forall k \in K \quad (6)$$

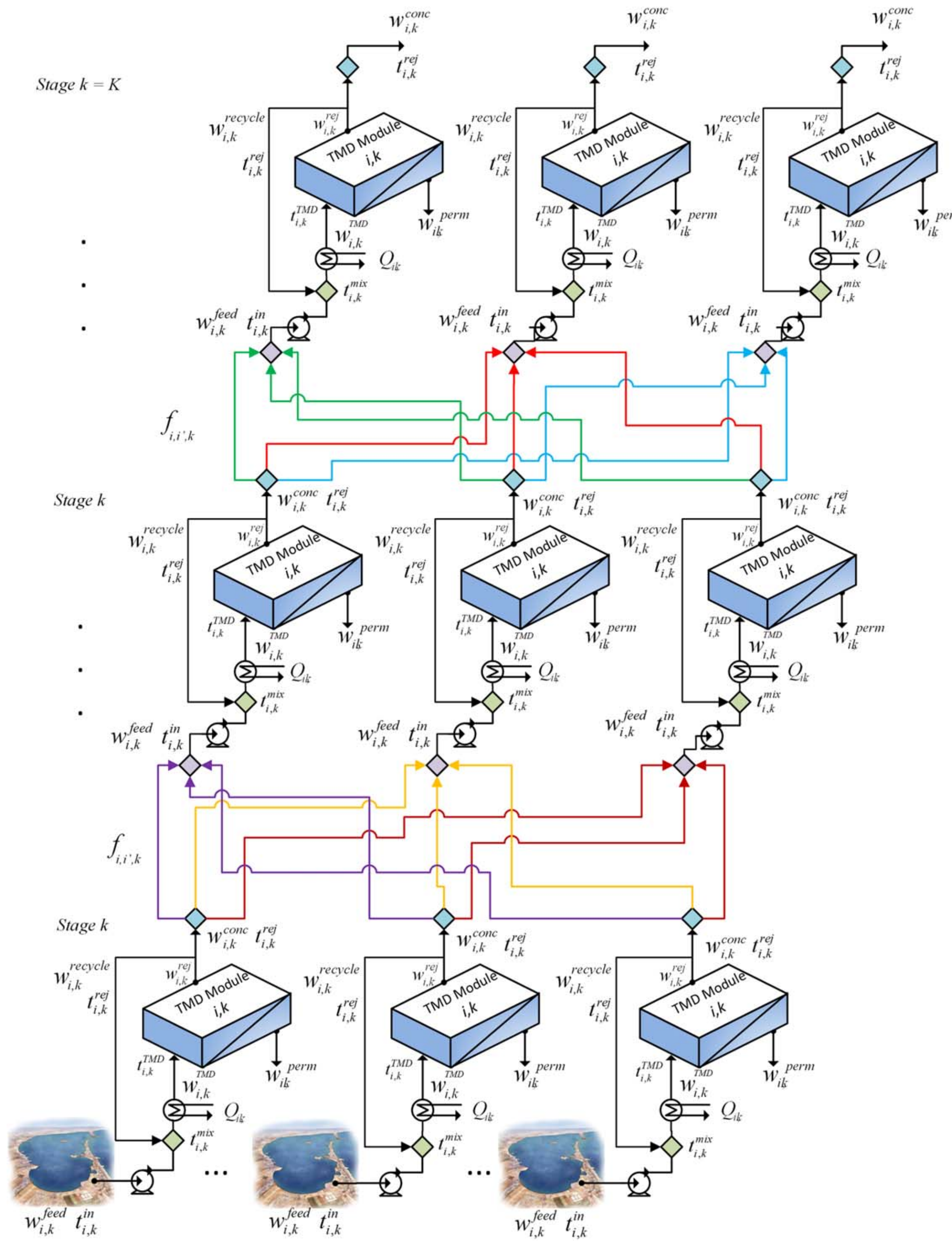

Figure 4. The proposed superstructure for synthesizing an integrated TMDN.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Mass balance around the recycle splitters

The stream that leaves the TMD unit ($w_{i,k}^{\text{rej}}$) is divided into two streams: $w_{i,k}^{\text{recycle}}$ that is returned to the main process and $w_{i,k}^{\text{conc}}$ that can be sent to another TMD unit

$$w_{i,k}^{\text{rej}} = w_{i,k}^{\text{recycle}} + w_{i,k}^{\text{conc}}, \forall i \in I, \forall k \in K \tag{7}$$

Notice that because of splitting, $w_{i,k}^{\text{recycle}}$ and $w_{i,k}^{\text{conc}}$ have the same concentration ($z_{i,k}^{\text{rej}}$), temperature ($t_{i,k}^{\text{rej}}$) and specific heat capacity ($\text{Cp}_{i,k}^{\text{rej}}$) as the original stream $w_{i,k}^{\text{rej}}$.

Mass balance for splitters

The concentrated water obtained from each TMD unit from each stage k ($w_{i,k}^{\text{conc}}$) is segregated to be sent to the next stage through the flows $f_{i,i',k}$

$$w_{i,k}^{\text{conc}} = \sum_{i'} f_{i,i',k}, \forall i \in I, \forall k \in K, k \neq \text{NOK} \tag{8}$$

Notice that previous relationship is not valid for the last stage of the superstructure because this concentrated water from the last stage is not used. In this case, NOK is the total number of stages. Also, $f_{i,i',k}$ has the same concentration ($z_{i,k}^{\mathrm{rej}}$), temperature ($t_{i,k}^{\mathrm{rej}}$), and specific heat capacity ($\mathrm{Cp}_{i,k}^{\mathrm{rej}}$) as the original stream $w_{i,k}^{\mathrm{conc}}$.

Mass balance, solute (or pollutant) balance, and energy balance for mixers

The sum of the concentrated water from the stage k from any TMD unit i ($f_{i,i',k}$) can be fed to the TMD units of the next stage ($w_{i',k+1}^{\mathrm{feed}}$)

$$\sum_i f_{i,i',k} = w_{i',k+1}^{\mathrm{feed}}, \forall i' \in I', \forall k \in K, k \neq \mathrm{NOK} \tag{9}$$

Previous relationship is not valid for the last stage because the last concentrated water is not fed to any other TMD unit.

The concentration in the mixer ($z_{i',k}^{\mathrm{feed}}$) is function of the concentration of the reject from the preceding stage from any unit i ($z_{i,k}^{\mathrm{rej}}$)

$$\sum_i z_{i,k}^{\mathrm{rej}} f_{i,i',k} = z_{i',k+1}^{\mathrm{feed}} w_{i',k+1}^{\mathrm{feed}}, \forall i' \in I', \forall k \in K, k \neq \mathrm{NOK} \tag{10}$$

To determine the temperature for the stream entering the TMD unit for stages other than the first, an energy balance is used for the mixer before any TMD unit. This way, the sum of the heat inlet from the streams from the previous stage ($\sum_i f_{i,i',k-1}\mathrm{Cp}_{i,k-1}^{\mathrm{rej}}(t_{i,k-1}^{\mathrm{rej}})$) is equal to the heat inlet to the TMD unit ($w_{i',k+1}^{\mathrm{feed}}\mathrm{Cp}_{i',k+1}^{\mathrm{feed}}(t_{i',k+1}^{\mathrm{in}})$) for each TMD unit i' in any stage k, and this is stated as follows

$$\begin{aligned} &\sum_i f_{i,i',k}\mathrm{Cp}_{i,k}^{\mathrm{rej}}(t_{i,k}^{\mathrm{rej}}) = w_{i',k+1}^{\mathrm{feed}}\mathrm{Cp}_{i',k+1}^{\mathrm{feed}}(t_{i',k+1}^{\mathrm{in}}), \\ &\forall i' \in I', \forall k \in K, k \neq \mathrm{NOK} \end{aligned} \tag{11}$$

where $\mathrm{Cp}_{i,k-1}^{\mathrm{rej}}$ and $\mathrm{Cp}_{i',k}^{\mathrm{feed}}$ are the heat capacities for streams of the previous stage and the one for the streams entering the TMD unit i' of the stage k, respectively. It should be noted that previous balance is not valid for the first stage because the inlet temperature is known for the feed of the network (e.g., seawater).

Design Equations for Each TMD Unit

For modeling the required TMD units (see Figure 4), the amount of permeated water is strongly influenced by the energy provided to the unit. In this context, the heating for each TMD unit is provided by external utilities and this is equal to the feed flow rate ($w_{i,k}^{\mathrm{TMD}}$) times the heat capacity for this stream ($\mathrm{Cp}_{i,k}^{\mathrm{TMD}}$) and the temperature difference ($t_{i,k}^{\mathrm{TMD}} - t_{i,k}^{\mathrm{mix}}$). Only a fraction of this added heat is used in vaporizing the permeate. The fraction is given by the efficiency factor ($\eta_{i,k}$). Hence, the heat balance for the TMD unit is given by

$$\eta_{i,k} Q_{i,k}^{\mathrm{heating}} = w_{i,k}^{\mathrm{perm}} \Delta h_{\mathrm{vw}_{i,k}}, \forall i \in I, \forall k \in K \tag{12}$$

where $w_{i,k}^{\mathrm{perm}}$ is the permeate flow rate and Δh_{vw} is the latent heat of vaporization. Experimental data may be used to measure the thermal efficiency. Alternatively, semiempirical expressions may be used. For instance, Elsayed et al. proposed the following expression for certain TMD modules (a polypropylene hollow-fiber membrane MD020CP2N manufactured by Microdyn)[13]

$$\eta_{i,k} = 1 - \frac{1.5 \cdot \frac{k_{\mathrm{m}_{i,k}}}{\delta}\left(t_{\mathrm{mfeed}_{i,k}} - t_{\mathrm{mperm}_{i,k}}\right)}{j_{\mathrm{w}_{i,k}} \Delta h_{\mathrm{vw}_{i,k}} + \frac{k_{\mathrm{m}_{i,k}}}{\delta}\left(t_{\mathrm{mfeed}_{i,k}} - t_{\mathrm{mperm}_{i,k}}\right)} \forall i \in I, \forall k \in K \tag{13}$$

where $k_{\mathrm{m}_{i,k}}$ is the thermal conductivity of the membrane, δ is the membrane thickness, $j_{\mathrm{w}_{i,k}}$ is the permeate flux passing through the membrane, $t_{\mathrm{mfeed}_{i,k}}$ is the temperature at the membrane in the feed side, and $t_{\mathrm{mperm}_{i,k}}$ is the temperature at the membrane in the permeate side. An example of an expression for calculating the thermal conductivity of a certain membrane is the correlation given by Elsayed et al.[13]

$$k_{\mathrm{m}_{i,k}} = 1.7 \times 10^{-7} t_{\mathrm{m}_{i,k}} - 4.0 \times 10^{-5} \ \forall i \in I, \forall k \in K \tag{14}$$

Flux modeling equations

The TMD unit is described using the modeling equations given by Elsayed et al.[13] Permeate flux passing through the membrane ($j_{\mathrm{w}_{i,k}}$) is calculated using the following expression

$$j_{\mathrm{w}_{i,k}} = b_{\mathrm{w}_{i,k}}\left(p_{\mathrm{wfeed}_{i,k}}^{\mathrm{vap}} \gamma_{\mathrm{wfeed}_{i,k}} x_{\mathrm{wfeed}_{i,k}} - p_{\mathrm{wperm}_{i,k}}^{\mathrm{vap}}\right), \forall i \in I, \forall k \in K \tag{15}$$

where $x_{\mathrm{wfeed}_{i,k}}$ is the mole fraction of water in feed, $p_{\mathrm{wfeed}_{i,k}}^{\mathrm{vap}}$ is the vapor pressure in the feed side, and $p_{\mathrm{wperm}_{i,k}}^{\mathrm{vap}}$ is the vapor pressure in the permeate side of the membrane, these vapor pressures are function of the temperature and these are calculated by the Antoine's equation as follows

$$P_{\mathrm{wfeed}_{i,k}}^{\mathrm{vap}} = \exp\left(23.1964 - \frac{3816.44}{t_{\mathrm{mfeed}_{i,k}} - 46.13}\right), \forall i \in I, \forall k \in K \tag{16}$$

$$P_{\mathrm{wperm}_{i,k}}^{\mathrm{vap}} = \exp\left(23.1964 - \frac{3816.44}{t_{\mathrm{mperm}_{i,k}} - 46.13}\right), \forall i \in I, \forall k \in K \tag{17}$$

$b_{\mathrm{w}_{i,k}}$ is a parameter for the molecular diffusion of water in air, and it can be calculated by multiplying the membrane permeability (B_{wB}) by the average temperature[34]

$$b_{\mathrm{w}_{i,k}} = B_{\mathrm{wB}} t_{m_{i,k}}^{1.334}, \forall i \in I, \forall k \in K \tag{18}$$

$t_{\mathrm{m}_{i,k}}$ is the average temperature in the TMD module determined as follows

$$t_{\mathrm{m}_{i,k}} = \frac{t_{i,k}^{\mathrm{TMD}} + t_{i,k}^{\mathrm{perm}}}{2}, \forall i \in I, \forall k \in K \tag{19}$$

$\gamma_{\mathrm{wfeed}_{i,k}}$ is the activity coefficient that is a function of the concentration. In the case of NaCl removal, it can be calculated as follows

$$\gamma_{\mathrm{wfeed}_{i,k}} = 1 - 0.5 x_{\mathrm{NaCl}_{i,k}} - 10 x_{\mathrm{NaCl}_{i,k}}^2, \forall i \in I, \forall k \in K \tag{20}$$

where $x_{\mathrm{NaCl}_{i,k}}$ is the mole fraction of NaCl in the feed.

Then, the molar fraction of NaCl in the feed ($x_{\mathrm{NaCl}_{i,k}}$) is function of the mass concentration in feed of the TMD unit ($z_{i,k}^{\mathrm{TMD}}$), the molecular weight of the water and the atomic weight of NaCl, and can be calculated with the next equation

$$\begin{aligned} x_{\mathrm{NaCl}_{i,k}} \cdot &\left[\left(z_{i,k}^{\mathrm{TMD}}/\mathrm{PM}_{NaCl}\right) + \left(1 - z_{i,k}^{\mathrm{TMD}}/\mathrm{PM}_{wt}\right)\right] \\ &= \left(z_{i,k}^{\mathrm{TMD}}/\mathrm{PM}_{NaCl}\right) \forall i \in I, \forall k \in K \end{aligned} \tag{21}$$

Finally, the mole fraction of the water in the feed is given as follows

$$x_{\text{wfeed}_{i,k}} = 1 - x_{\text{NaCl}_{i,k}} \forall i \in I, \forall k \in K \tag{22}$$

Membrane area and temperature profile

The membrane area can be calculated by dividing the permeate water flow rate ($w_{i,k}^{\text{perm}}$) by the water flux

$$A_{\text{m}_{i,k}} = \frac{w_{i,k}^{\text{perm}}}{j_{\text{w}_{i,k}}}, \forall i \in I, \forall k \in K \tag{23}$$

The temperature profile in the membrane can be calculated through the use of the temperature polarization coefficient given by the next equation[35]

$$\theta_{i,k} = \frac{t_{\text{mfeed}_{i,k}} - t_{\text{mperm}_{i,k}}}{t_{i,k}^{\text{TMD}} - t_{i,k}^{\text{perm}}}, \forall i \in I, \forall k \in K \tag{24}$$

where $\theta_{i,k}$ is the temperature polarization coefficient and $t_{i,k}^{\text{TMD}}$ is the temperature of the feed in the bulk in the feed side, $t_{\text{mfeed}_{i,k}}$ is the temperature of the feed at the membrane, $t_{i,k}^{\text{perm}}$ is the temperature of permeate in the bulk, $t_{\text{mperm}_{i,k}}$ is the temperature of permeate at the membrane, as shown in Figure 5.

The temperature polarization coefficient ($\theta_{i,k}$) may be evaluated from experimental data. Alternatively, correlations may be used for certain membranes. For instance, Elsayed et al. showed a linear behavior as a function of the temperature according to the next expression[13]

$$\theta_{i,k} = 1.104 - 0.00086 \cdot t_{i,k}^{\text{TMD}} \quad \forall i \in I, \forall k \in K \tag{25}$$

The temperature for the reject stream ($t_{i,k}^{\text{rej}}$) is an average between the temperature in the bulk ($t_{i,k}^{\text{TMD}}$) and the temperature at the membrane boundary layer ($t_{\text{mfeed}_{i,k}}$), as shows in the next expression

$$t_{i,k}^{\text{rej}} = \frac{t_{i,k}^{\text{TMD}} + t_{\text{mfeed}_{i,k}}}{2} \quad \forall i \in I, \forall k \in K \tag{26}$$

The vaporization of water ($\Delta h_{\text{vw}_{i,k}}$) in the feed side is calculated from the next correlation[13]

$$\Delta h_{\text{vw}_{i,k}} = 3190 - 2.5009 t_{\text{mfeed}_{i,k}} \quad \forall i \in I, \forall k \in K \tag{27}$$

Logical relationships

Logical relationships are required to determine if a potential unit from the superstructure is required in the optimal solution. This depends on the amount of water that is recovered, the final concentration of permeate, and the impact on the total cost (capital and operating). Therefore, binary variables are used to indicate the existence or absence of such units (i.e., the binary variable associated to a given TMD unit is one when the unit exists, otherwise it is zero, $y_{i,k} = \text{Unitexistance}, \{0, 1\}$). This is modeled through the following relationship:

$$W^{\min} \cdot y_{i,k} \leq w_{i,k}^{\text{feed}} \leq W^{\max} \cdot y_{i,k}, \forall i \in I, \forall k \in K \tag{28}$$

where $W^{\max}$ and $W^{\min}$ are upper and lower limits for the flow rate of the feed that can be used in a TMD unit. When the inlet flow rate to the unit i in the stage k ($w_{i,k}^{\text{feed}}$) is greater than zero then the associated binary variable ($y_{i,k}$) must be one. Conversely, when the binary variable $y_{i,k}$ is zero (i.e., the associated TMD unit does not exist), the treated flow rate ($w_{i,k}^{\text{feed}}$) must be zero.

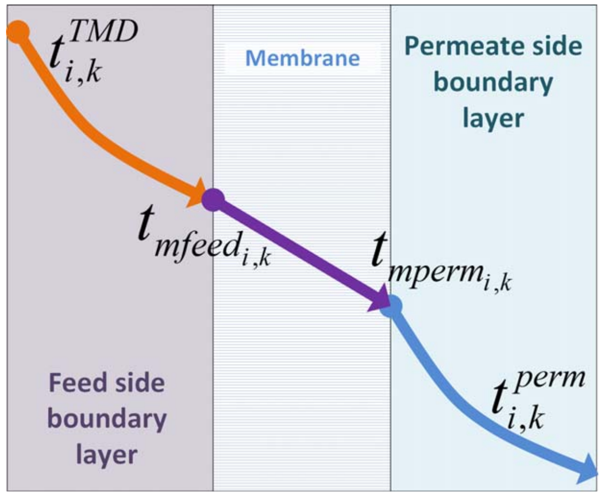

Figure 5. Temperature profile for the boundary layers and the membrane.[36]

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Total feed water

The total feed (or total raw water) "TRW" fed to the TMDN is the sum of the feed streams entering stage 1 in any unit i. Therefore

$$\text{TRW} = \sum_i w_{i,1}^{\text{feed}} \tag{29}$$

Hence, the maximum and minimum amount of water that can be supplied to the entire system is stated as follows

$$\text{TRW}^{\min} \leq \text{TRW} \leq \text{TRW}^{\max} \tag{30}$$

Total permeated water

The total permeated water (TPW) is equal to the sum of the permeated water from all units i of any stage k ($w_{i,k}^{\text{perm}}$)

$$\text{TPW} = \sum_i \sum_k w_{i,k}^{\text{perm}} \tag{31}$$

Restriction for the maximum amount of water

The maximum and minimum amount of permeate is bounded as follows

$$\text{TPW}^{\min} \leq \text{TPW} \leq \text{TPW}^{\max} \tag{32}$$

where $\text{TPW}^{\min}$ and $\text{TPW}^{\max}$ are the lower and upper limits for the demanded clean permeate.

Restriction of the concentration in the reject (or brine)

The maximum concentration allowable in the reject must meet the following constraint

$$z_{i,k}^{\text{rej}} \leq z^{\max} \cdot y_{i,k} \forall i \in I, \forall k \in K \tag{33}$$

$z^{\max}$ is multiplied by the binary variable $y_{i,k}$ because this constraint only applies when the unit exists.

Total heat consumed

The total heat consumed in the network (THeat) accounts for the heat consumed in each stage of the superstructure for each TMD unit ($Q_{i,k}^{\text{Heating}}$) as follows

Table 1. Data for the Examples Presented[13]

Parameter	Units	Value
$A_{\rm m}^{\max}$·=·Maximum permissible area in TMD unit	M2	100 (for Case Study 1) and 30 for Case Study 2
$B_{\rm wB}$·=·Temperature independent base value for the permeability	kg/(m2· s ·Pa ·K1.334)	7.5 ×x 10−−8
$C_{\rm F1}^{\rm TMD}$·=·Fixed cost of the membrane-related units	\$/m2	58.5
$C_{\rm F2}^{\rm TMD}$·=·Fixed cost of the non-membrane units	\$/(kg/s)	1,115
$C_{\rm inst}^{\rm TMD}$·=·Installation cost of the TMD module	\$/m2	25% of the purchase cost
$C_{\rm Op1}^{\rm TMD}$·=·Operational cost related to the TMD module	\$	1,411
$C_{\rm Op2}^{\rm TMD}$·=·Operational cost related to the reject processing	\$	43
$C_{\rm Op3}^{\rm TMD}$·=·Operational cost related to the feed treatment	\$	1,613
$z^{\max}$·=·Maximum permissible concentration	wt %	0.5
$\beta_k^{\rm TMD}$·=·Exponent for area cost of the modules	—	1.0
$\xi_{i,k}$·=·Fractional recovery of permeate in TMD unit	—	0.8
δ·=·Membrane thickness	mm	0.00065

$$\mathrm{THeat}=\sum_i\sum_k Q_{i.k}^{\rm Heating} \tag{34}$$

Total membrane area

The total membrane area needed in the desalination units (TMA) is the sum of the areas in each stage in the superstructure for each TMD unit ($A_{\mathrm{m}_{i,k}}$) as follows

$$\mathrm{TMA}=\sum_i\sum_k A_{\mathrm{m}_{i,k}} \tag{35}$$

where the maximum area in the TMD units is bounded by the maximum available size of a TMD unit as follows

$$A_{\mathrm{m}_{i,k}}\le A_{\rm m}^{\max}\quad \forall i\in I,\forall k\in K \tag{36}$$

Initial data

The concentration in the first stage of the superstructure ($z_{i,1}^{feed}$) is the concentration of the water to be treated. This is a known value for any TMD unit in stage 1. For instance, in the case of the raw feed being seawater, the constraint is given by

$$z_{i,1}^{\rm feed}=z^{\rm seawater}\quad \forall i\in I \tag{37}$$

Also, the temperature in the first stage for the raw feed water ($t_{i,1}^{\rm in}$) is a known value and this is valid for any TMD unit in stage 1, $t_{i,1}^{\rm in}$ depends on the treated feed. For instance, in the case when the feed is at ambient temperature, the constraint is given by

$$t_{i,1}^{\rm in}=\mathrm{T}^{\rm amb}\quad \forall i\in I \tag{38}$$

Objective function

The total annual cost for the TMDN takes into account the fixed cost for the TMD units as a function of the membrane area ($C_{\rm F1}^{\rm TMD}$) for TMD modules and as a function of the flow rate of the stream to be fed ($C_{\rm F2}^{\rm TMD}$) for nonmembrane elements, the installation costs of the TMD unit ($C_{\rm inst}^{\rm TMD}$), and the operating cost for the TMD units ($C_{\rm Op1}^{\rm TMD}$; $C_{\rm Op2}^{\rm TMD}$; $C_{\rm Op3}^{\rm TMD}$). The exponent $\beta_k^{\rm TMD}$ is a factor used to account for the economy of scale. The terms $\mathrm{Cost}^{\rm Heating}$ and $\mathrm{Cost}^{\rm Cooling}$ are the unit cots for heating and cooling utilities, respectively. Additionally, $k_{\rm F}$ is the factor used to annualize the investment and $H_{\rm Y}$ is a factor used to account the operating time per year. It is worth noting that the fixed charge for the involved units is only considered when the units exist through the use of the binary variables ($y_{i,k}$). Thus, total annual cost is stated as follows

$$\mathrm{TAC}=\left[\begin{array}{l} k_{\rm F}\sum_i\sum_k\left[\left(C_{\rm inst}^{\rm TMD}\cdot y_{i,k}\right)+\left(C_{F1}^{TMD}\cdot A_{\mathrm{m}_{i,k}}\right)^{\beta_2}+\left(C_{F2}^{TMD}\cdot w_{i,k}^{\rm TMD}\right)^{\beta_3}\right]\\ +H_{\rm Y}\sum_i\sum_k\left[C_{Op1}^{TMD}\cdot y_{i,k}+C_{Op2}^{TMD}\cdot\left(1-\xi_{i,k}\right)w_{i,k}^{\rm feed}+C_{Op3}^{TMD}\cdot w_{i,k}^{\rm TMD}\right]\\ +H_{\rm Y}\sum_i\sum_k\left[\mathrm{Cost}^{\rm Heating}\cdot Q_{i,k}^{\rm Heating}\right]\end{array}\right] \tag{39}$$

where the fraction of recovery ($\xi_{i,k}$) can be obtained by dividing the permeate water flow rate ($w_{i,k}^{\rm perm}$) by the feed water flow rate at each stage

$$\xi_{i,k}=\frac{w_{i,k}^{\rm perm}}{w_{i,k}^{\rm feed}}\quad \forall i\in I,\forall k\in K \tag{40}$$

The annual gross profit (AGP) can be calculated as the annual permeate value (APV) minus the total annual cost, according to the following equation

$$\mathrm{AGP}=\mathrm{APV}-\mathrm{TAC} \tag{41}$$

Case Studies

Two case studies are presented to show the applicability of the proposed approach for designing TMDN, the first one corresponds to a seawater desalination process whereas the second one corresponds to treating wastewater from the syrup manufacturing process. Table 1 shows key information for the two case studies.[13] The optimization formulation was developed based on the aforementioned mathematical problem. The problem was coded using the software GAMS,[37] where the solver DICOPT was used to solve the associated

Table 2. Problem Statistics

Item	Value
Number of continuous variables	1,030
Number of binary variables	25
Number of constraints	964
CPU time (s)	5.56

Table 3. Data for Case Study 1

Concept	Unit	Value
General Data: Desalination[13]		
Heating cost	\$/10^9 J heating utility	5.00
Cooling cost	\$/10^9 J cooling water at 293 K	4.00
Pumping	\$/m^3	0.056
Labor	\$/m^3	0.030
Cost of permeate	\$/m^3	8.00
Initial flow rate	kg/h	3,456
Initial temperature	K	300
Initial concentration	% weight	0.1
Annual operation	h	8,000
Membrane Specifications[13,38]		
Membrane thickness	mm	0.65
Membrane cost	\$/m^2	90
Maximum membrane area per module	m^2	100
Membrane life time	year	4

mixed-integer nonlinear programing. To solve the problem, a computer with an Intel® Core TM i7–4700MQ processor at 2.40 GHz and 8 GB of RAM was used. The problem and solution statistics are shown in Table 2.

Case Study 1: Seawater desalination process

Worldwide fresh water demand is rising, largely driven by the increase in the population and living standards, seawater and brackish water desalination has become an alternative for new water supply in coastal areas, especially in areas with stressed and overdrawn fresh water resources. The proposed optimization model described in Optimization Model for TMDN was used to synthesize a TMDN for brackish water desalination on the coast of Saudi Arabia. The feed has 0.1% (1000 ppm) dissolved NaCl and an initial flow rate of 3456 kg/h. The size of each module is considered with a maximum membrane area of 100 m^2/module. The objective function consists of maximizing the annual gross profit obtained from the sales of permeate minus the total annualized cost of the system. The problem data are given in Table 3.[13,38]

The optimal solution obtained for the case study is presented as the solution of the first scenario. In addition, nine additional configurations were analyzed to show the advantages of the proposed approach.

The optimal solution (first scenario) is shown by Figure 6. It involves two units in series. A total recovery of 95.90% of water as permeate was obtained. The feed temperature for the first unit (1,1) is 300 K and the feed temperature for the second unit (1,2) is 359 K. The first unit has a membrane area of 92.4 m^2 which represents 83.2% of the total required area of membrane and produces a permeate flow rate of 2764 kg/h which is 83.4% of the total permeate of the TMDN. The total permeate is 3316 kg/h, which yields an annual sales revenue of \$212,340/year. The total annual cost is \$170,347/year and the annual gross profit for the optimal solution is \$41,993/year. Unit cost for optimal solution is 6.3 \$/m^3, which is relatively high, however, Elsayed et al. have shown that the unit costs can be substantially reduced when the desalination process is coupled with industrial facilities for heat recovering.[13] Table 4 shows the main economic results for the optimal configuration and for other analyzed scenarios.

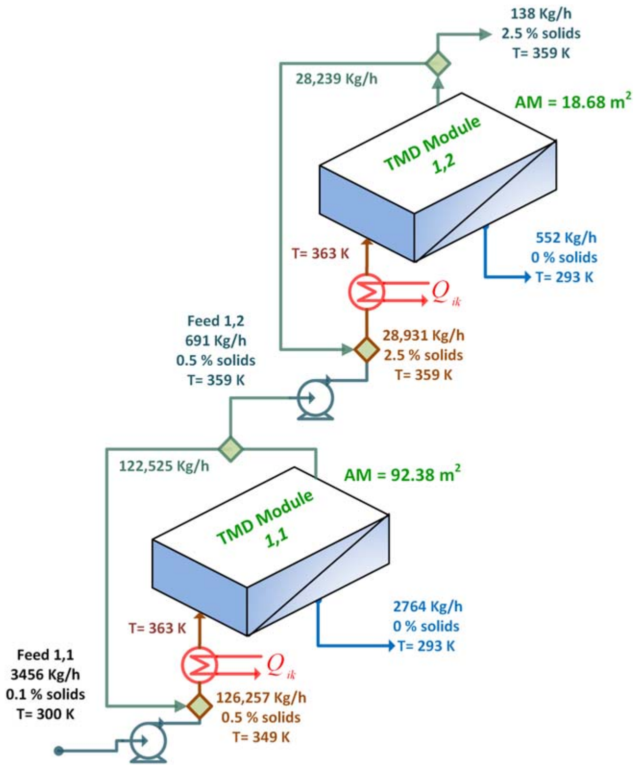

Figure 6. Optimal TMDN for Case Study 1.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

To show the merits of the optimal solution (Scenario 1), nine other scenarios are synthesized, analyzed, and compared with the optimal solution. These scenarios are shown by Figures 7–11. Also, the results of these scenarios are shown in Table 4.

Figure 7a shows the Scenario 2 considering only one TMD unit, for this case the total heating cost is \$31,355/

Table 4. Optimal Results for Case Study 1 (Scenario 1) and Comparison with 9 Nine Other Scenarios

		Scenario								
Concept	Unit	Optimal	2	3	4	5	6	7	8	9
Total membrane area	m2	111.06	92.38	92.38	92.37	115.09	111.03	111.02	111.02	115.05
Number of TMD units	—	2	1	2	3	3	3	4	4	6
Thermal efficiency	—	0.900	0.902	0.902	0.902	0.890	0.900	0.900	0.900	0.880
Total permeate	kg/h	3,316	2,764	2,764	2,763	3,426	3,316	3,315	3,316	3,425
Total feed water	kg/h	3,456	3,456	3,456	3,456	3,456	3,456	3,456	3,456	3,456
Total recovery	%	95.90	80.00	79.97	79.94	98.00	96.00	95.92	96.40	98.60
Total heating cost	\$/year	37,633	31,355	31,360	31,344	38,875	37,632	37,616	37,632	38,864
Total annual cost	\$/year	170,347	136,644	142,330	146,742	180,665	173,780	178,192	180,398	195,126
Permeate value	\$/year	212,340	176,950	176,948	176,948	219,400	212,275	212,275	212,275	219,335
Annual profit	\$/year	41,993	40,306	34,617	30,205	38,735	38,495	34,083	31,877	24,209
Unit cost	\$/m3	6.3	6.1	6.4	6.6	6.6	6.5	6.7	6.8	7.1

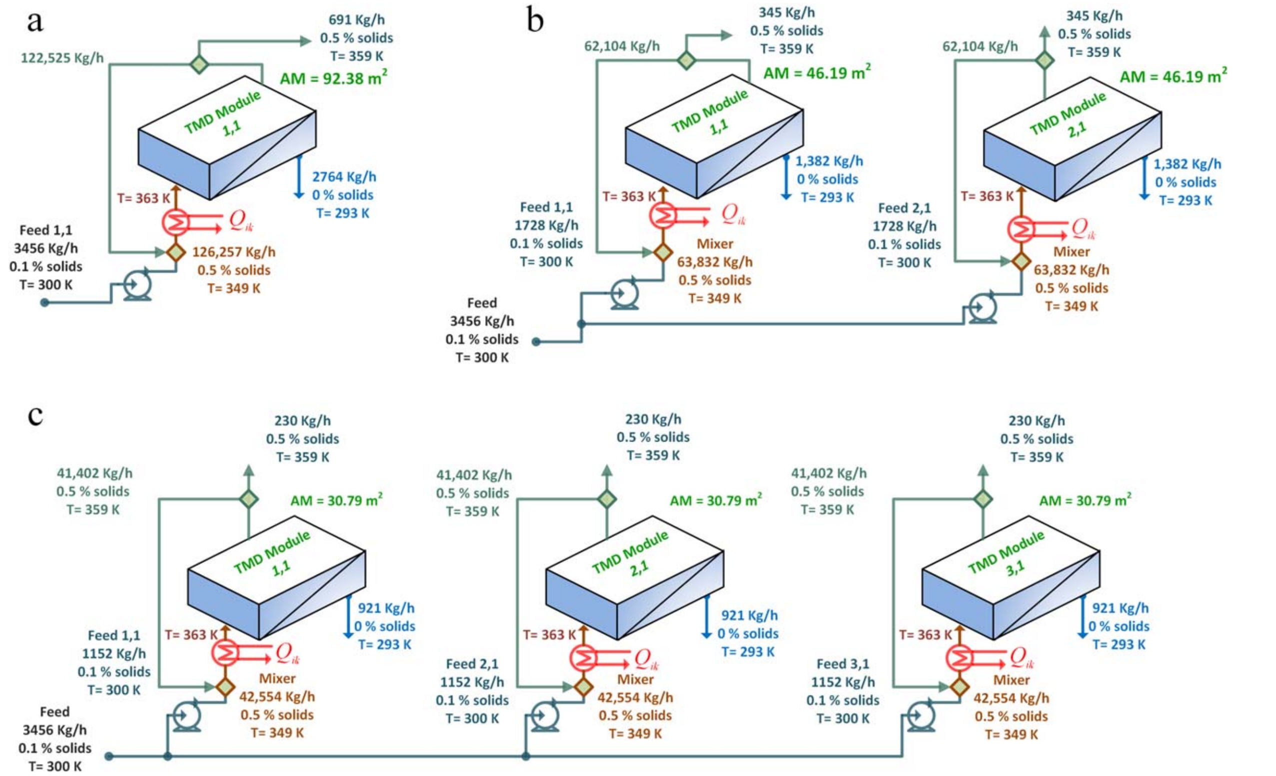

Figure 7. TMDN for Case Study 1: (a) Scenario 2, (b) Scenario 3, and (c) Scenario 4.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

year, the total permeate is 2764 Kg/h which has an annual value of \$176,950/year. The annual gross profit for this case is \$40,306/year which is 4% less than the optimal solution. Figure 7b presents the solution of Scenario 3 considering two TMD units in parallel arrangement. The total heating cost is \$31,360/year and the total permeate is 2764 kg/h with an

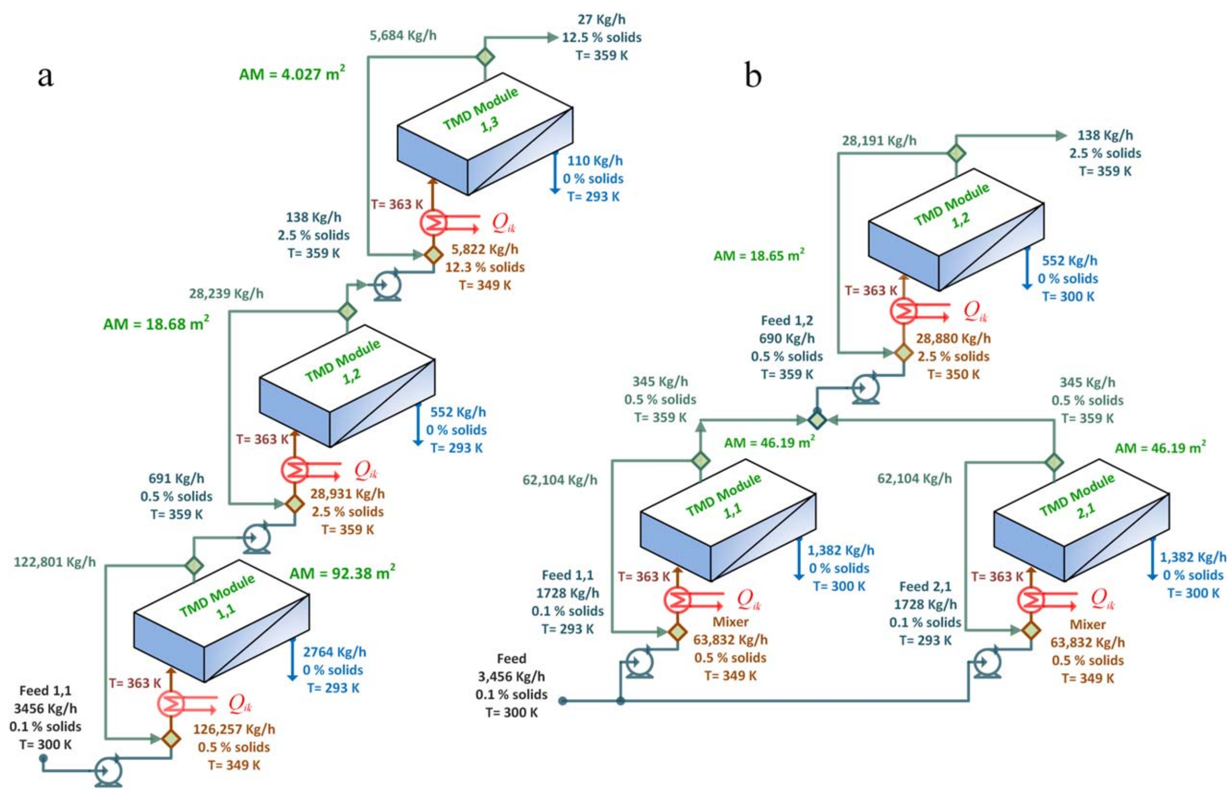

Figure 8. TMDN for Case Study 1: (a) Scenario 5 and (b) Scenario 6.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

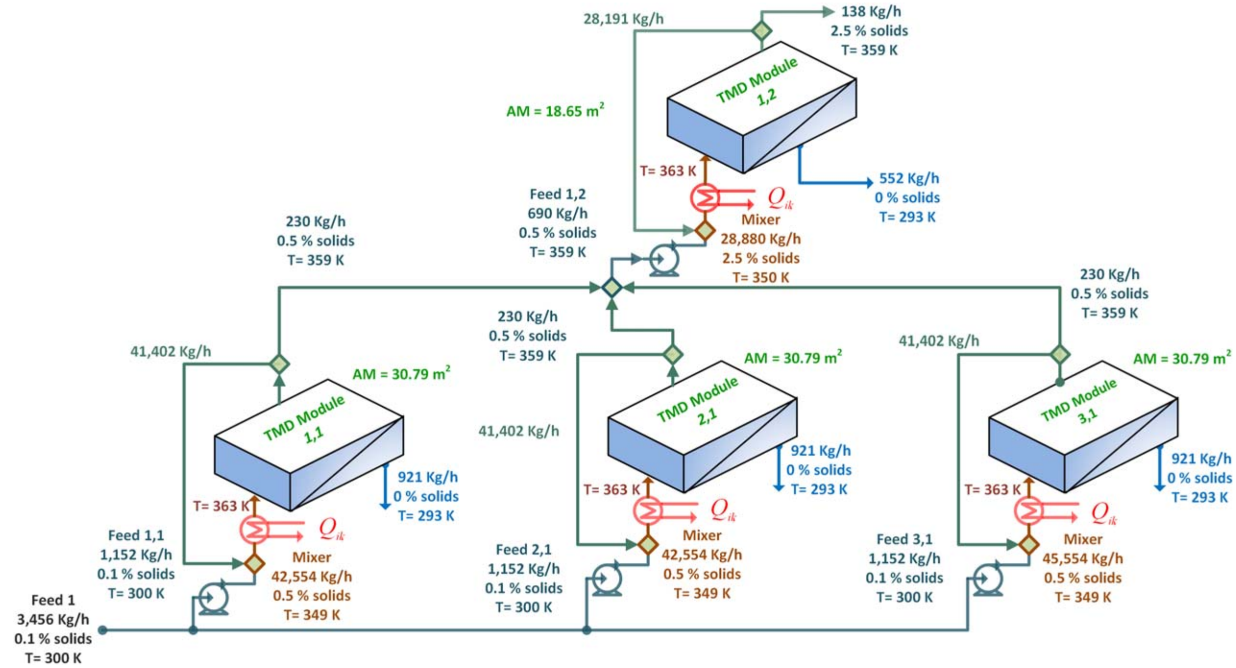

Figure 9. TMDN for solution of Scenario 7 for Case Study 1.

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annual value of \$176,948/year. The total annual cost for this scenario is \$142,330/year and the annual gross profit is \$34,617/year which is 17.6% less than the optimal solution. Scenario 4 (see Figure 7c) shows three TMD units ordered in parallel arrangement, this change increases the total annual costs up to \$146,742/year, which yields an annual gross profit of \$30,205/year that is 28.1% less than the optimal solution.

Scenario 5 presents three units in series arrangement as shown in Figure 8a, for this case, the total annual cost is \$180,665/year yielding an annual gross profit \$38,735/year (7.76% less than the optimal solution). Notice that the total annual cost increases even though the recovery of permeate is higher, this is because of the cost associated with the installation and operation of the three TMD units in series

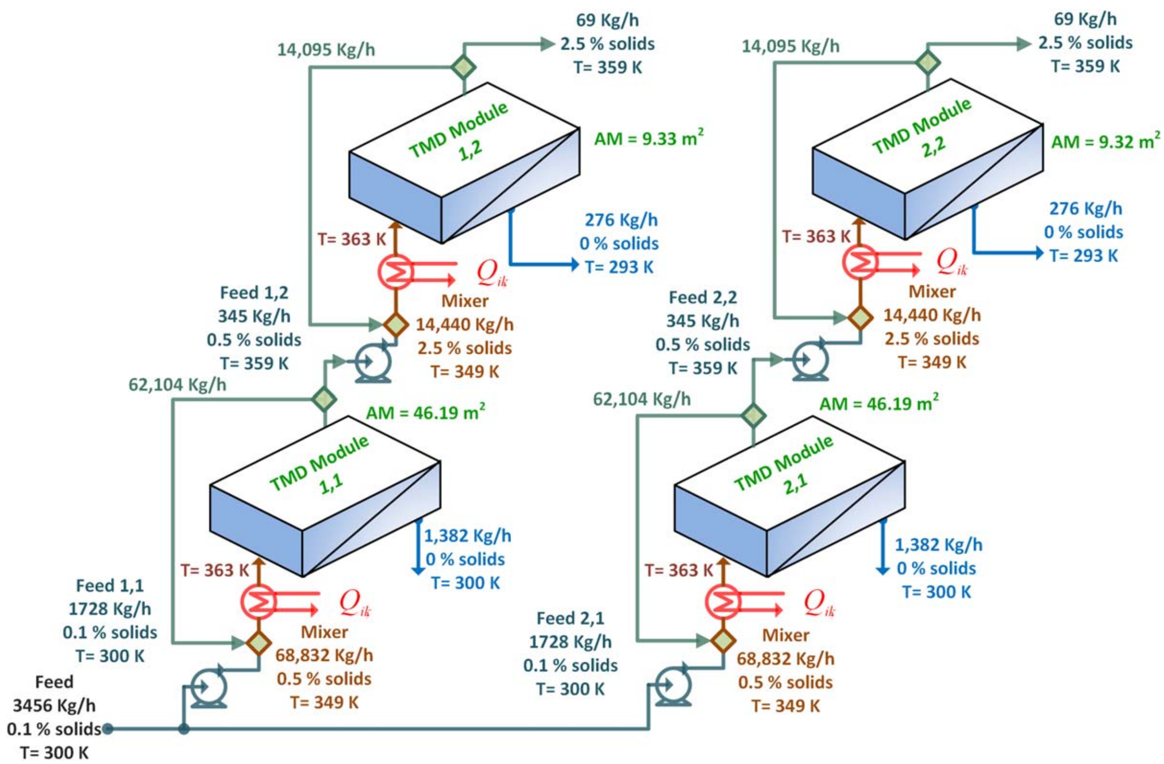

Figure 10. TMDN for solution of Scenario 8 of Case Study 1.

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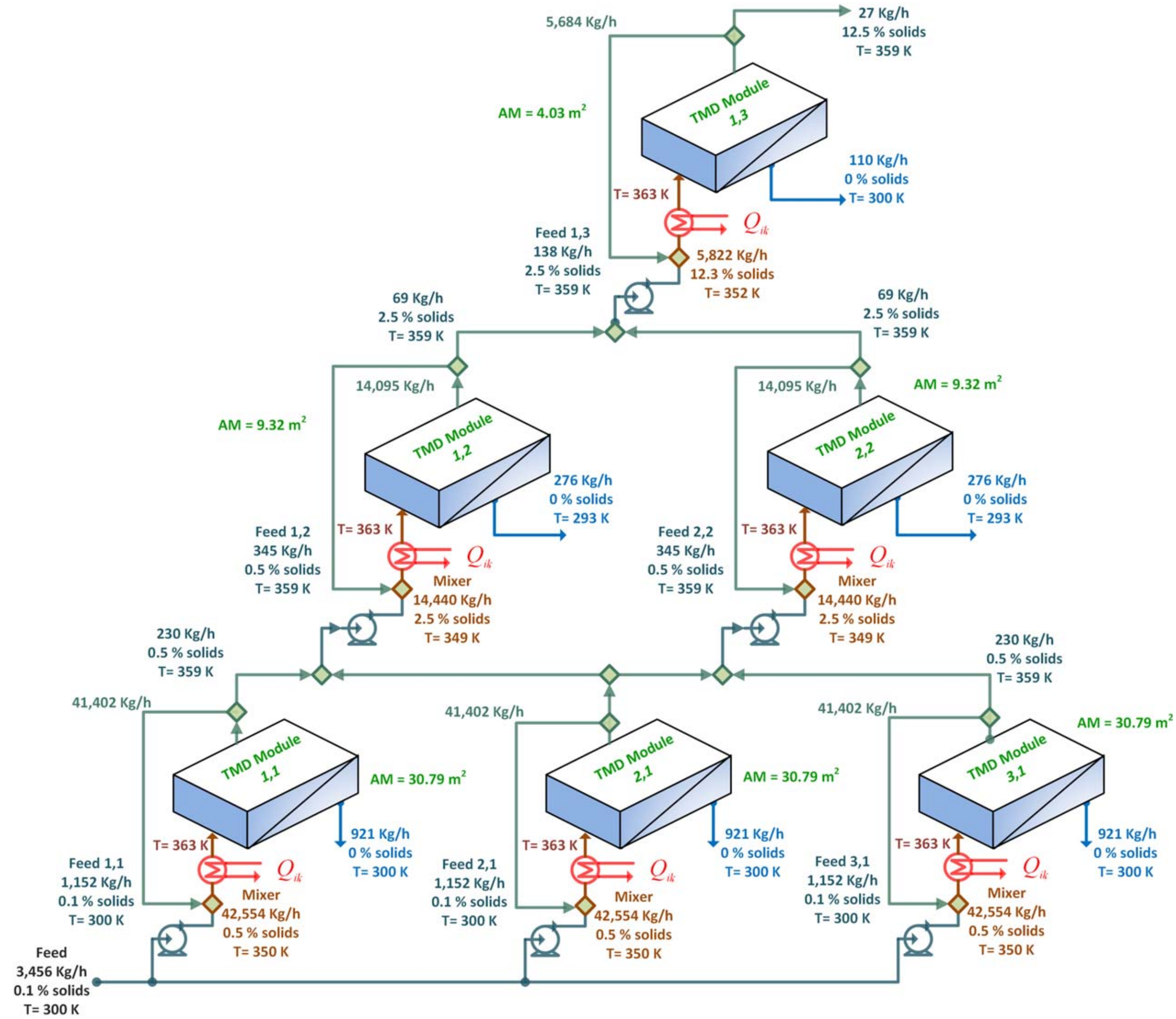

Figure 11. TMDN for solution of Scenario 9 for Case Study 1.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

arrangement. Figure 8b presents a hybrid configuration between two TMD units in parallel arrangement and one unit in series arrangement, the total heating cost is $37,632/year and the total permeate is 3316 kg/h with an annual value of $212,275/year. The total annual cost for this Scenario 6 is $173,780/year and the annual gross profit is $38,495/year which is 8.3% less than the optimal solution.

Scenario 7 is presented as a hybrid arrangement of three units in parallel and one unit in series that receives the concentrated from the previous stages (see Figure 9). The total annual costs are $178,192/year, which yields an annual gross profit of $34,083/year that represents 18.8% less than the optimal solution. In Scenario 8, two modules in parallel arrangement and two units in series arrangement are used (see Figure 10), having a total annual cost of $180,398/year, where the annual gross profit is $31,877/year which is 24.1% less than the optimal solution.

Finally, Scenario 9 presents three TMD units in parallel arrangement in the first stage, two units in parallel arrangement in the second stage connected in series arrangement with the units of the previous stage and one unit in the third stage that receives the concentrate of the units of the previous stage as shown in Figure 11. For this Scenario 9, the total heating cost is $38,864/year and the total permeate is 3425 kg/h with an annual value of $219,335/year. The total annual cost for this Scenario 9 is $195,126/year and the annual gross profit is $24,209/year which is 42.4% less than the optimal solution.

Case Study 2: Syrup concentration process

TMD technology can be employed in the dextrose syrup manufacturing process for the partial concentration of dextrose syrup. Laboratory tests indicate that water can be removed from the solution up to the extent of 55% removal. This translates to syrup of 11% concentration. Also, permeate was found to contain only trace quantities of sugars which allows additional usage of permeate. As the specifications of commercial sugar syrup dictate a dextrose concentration of about 66%, conventional evaporation is used for the rest of the concentration task. The flow sheet for the syrup production process proposed by Silayo et al. is shown in Figure 12a.[39] The objective of this case study is to consider the use of a TMDN for the proposed production process of syrup concentration in conjunction with the evaporator–condenser scheme. A TMDN was synthesized with stream S7 of the system as the feed to the network. The hybrid separation network for concentration

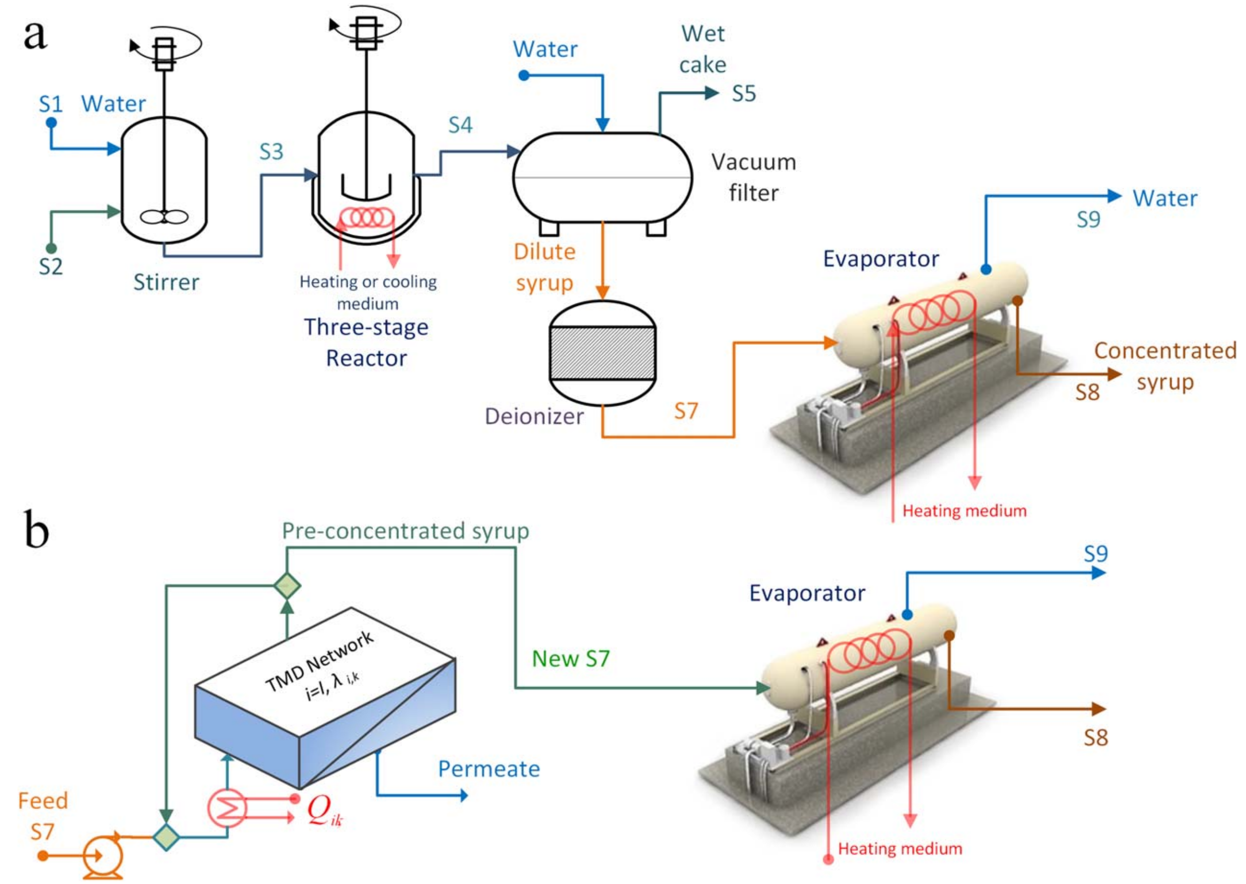

Figure 12. (a) Dextrose syrup production process [39] and (b) hybrid TMD-evaporation system for the concentration of dextrose syrup.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

of syrup is shown in Figure 12b. This hybrid separation scheme leads to significant savings in energy costs. The feed has 5% (50,000 ppm) of dextrose and an initial flow rate of 648 kg/h. The size of each module is constrained by a maximum membrane area of 30 m^2/module. The problem data are given in Table 5.[13,38]

Table 5. Data for Case Study 2

Concept	Unit	Value
General Data: Syrup Concentration[13]Concept		
Heating cost	$/10^9 J heating utility	5.00
Cooling cost	$/10^9 J cooling water at 293 K	4.00
Pumping	$/m^3	0.056
Labor	$/m^3	0.030
Cost of permeate	$/m^3	8.00
Feed flow rate	kg/h	648
Initial temperature	K	300
Initial concentration	% weight	5
Annual operation	h	8,000
Membrane Specifications[13,38]		
Membrane thickness	mm	0.65
Maximum membrane temperature	K	363
Membrane cost	$/m^2	90
Maximum membrane area per module	m^2	30
Membrane life time	year	4

The optimal solution for this Case Study 2 for the syrup concentration process is represented as Scenario 1. For comparison, two additional scenarios are presented.

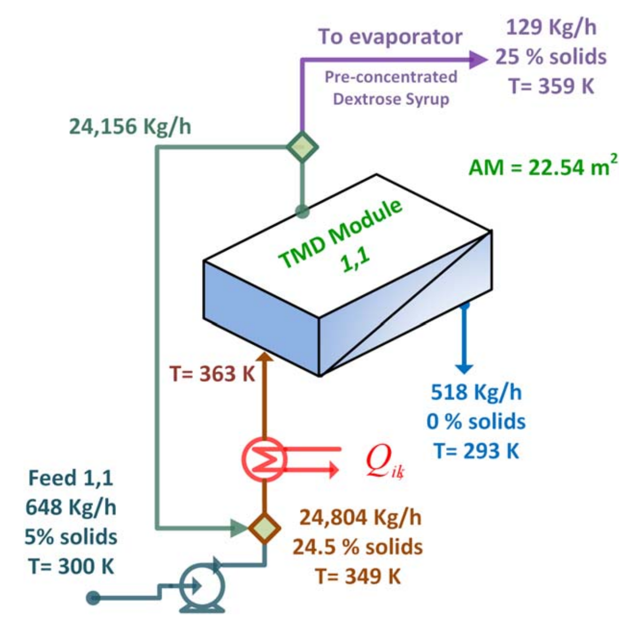

Figure 13. TMDN for optimal solution of Scenario 1 of Case Study 2.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Table 6. Results for Syrup Concentration

Concept	Unit	Scenario 1	Scenario 2	Scenario 3
Total membrane area	m^2	22.542	22.542	22.56
Number of TMD units	–	1	2	3
Thermal efficiency	–	0.879	0.874	0.873
Total permeate	kg/h	518	518	518
Total feed water	kg/h	648	648	648
Total heating cost	\$/y	6,064	6,064	6,063
Total annual cost	\$/y	30,589	35,000	39,411
Permeate value	\$/y	33,178	33,178	33,178
Annual profit	\$/y	2,589	−-1,822	−-6,233
Unit cost	\$/m^3	7.3	8.4	9.5

The optimal solution for the Case Study 2 represented as Scenario 1 is shown by Figure 13. For this case, the thermal efficiency is 0.874, the optimal TMD temperature is 363 K, the total recovered permeate is 79%, and the feed temperature is 300 K. Notice that in the optimal solution there is no requirement for serial staging because the concentration increases at the end of the first stage at a level higher than the maximum allowable into the membrane. The membrane area for Scenario 1 is 22.542 m^2, the total heating cost is \$6,064/year, and the permeate flow rate is 518 kg/h, which has a value for permeate of \$33,178/year. The total annual cost is \$30,589/year and the annual gross profit for the optimal solution is \$2,589/year. Table 6 shows the main economic results for the optimal configuration and also for the two other scenarios represented by Figure 14.

Scenarios 2 (see Figure 14a) and 3 (see Figure 14b) have analogous results to Scenario 1, but there are differences in configuration and in the total costs. Scenario 2 presents two TMD units ordered in parallel arrangement, with a total annual cost of \$35,000/year, which yields economic losses of \$1822/year. Conversely, Scenario 3 presents three TMD units in parallel arrangement, increasing the total annual cost to \$39,411/year and economic losses of \$6233/year.

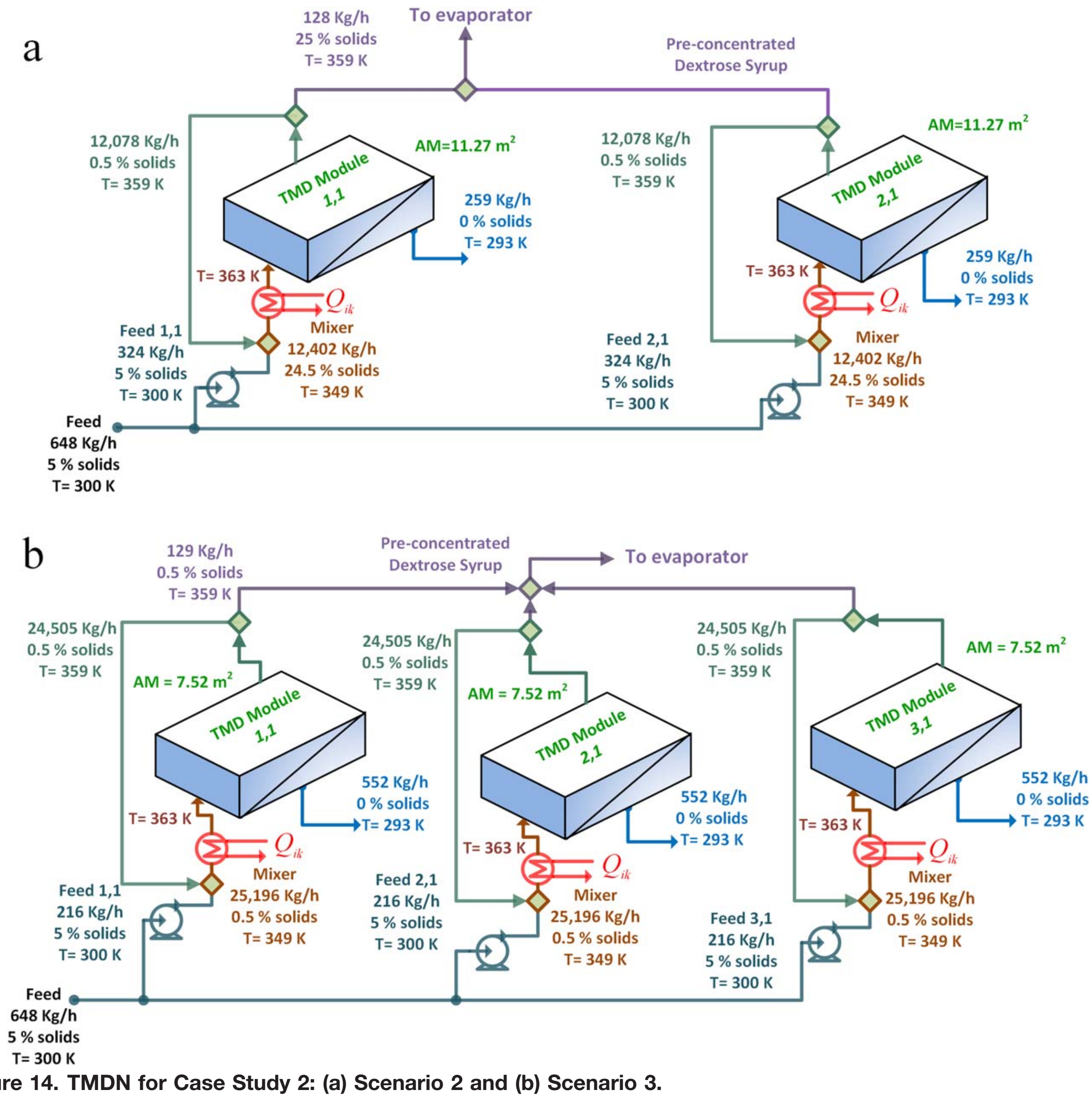

Figure 14. TMDN for Case Study 2: (a) Scenario 2 and (b) Scenario 3.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Conclusions

This article has presented an optimization approach for synthesizing a TMDN. A superstructure has been developed to embed network configurations of interest. This superstructure allows various arrangements of TMD modules, pumps, heaters, and condensers. It also accounts for stream recycle and assignment. An optimization formulation has been developed. The proposed model incorporates modeling equations as well as technical and design constraints. The model has been formulated as a mixed-integer nonlinear programming model. The proposed optimization model was applied to two case studies, where the optimal network structure as well as the operating conditions were determined. The obtained results show that the proposed model yields better results than other configurations. The results show that the main limitation of the TMD is the high cost for obtaining purified water, this is mainly because the high energy costs. However, one of the important features of TMD units is that these require low temperature heat, therefore, one future work must consider the simultaneous energy integration with processes for waste heat recovery, which can be very useful for reducing the unit cost of the clean water obtained.

Acknowledgment

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Notation

Indexes

i = units connected in parallel
i' = subsequent line for i
k = units connected in series

Sets

I = set for units connected in parallel
I' = set for subsequent line of units connected
K = set for units connected in series

Parameters

$A_{\mathrm{m}}^{\max}$ = maximum permissible area in TMD unit, m^2
B_{wB} = temperature independent base value for the permeability, kg/(m^2 s Pa K$^{1.334}$)
$C_{\mathrm{F1}}^{\mathrm{TMD}}$ = fixed cost of the TMD module, \$/m^2
$C_{\mathrm{F2}}^{\mathrm{TMD}}$ = fixed cost of the TMD module, \$/(kg/s)
$C_{\mathrm{inst}}^{\mathrm{TMD}}$ = installation cost of the TMD module, \$/m^2
$C_{\mathrm{Op}}^{\mathrm{TMD}}$ = operational cost of the TMD module, \$
$\mathrm{Cost}^{\mathrm{Heating}}$ = cost of heat utility, \$/kW
$\mathrm{Cost}^{\mathrm{Cooling}}$ = cost of cold utility, \$/kW
$\mathrm{Cp}_{i,k}^{\mathrm{feed}}$ = specific heat capacity for the feed stream, kJ/(kg K)
$\mathrm{Cp}_{i,k}^{rej}$ = specific heat capacity for the rejected stream, kJ/(kg K)
$\mathrm{Cp}_{i,k}^{TMD}$ = specific heat capacity for the stream fed the TMD unit, kJ/(kg K)
H_{Y} = operating hours for the plant per year
K_{F} = factor used to annualize the capital cost
PM_{NaCl} = molecular weight of NaCl
PM_{wt} = molecular weight of water
$\mathrm{TPW}^{\min}$ = minimum amount of total permeate water, kg/s
$\mathrm{TPW}^{\max}$ = maximum amount of total permeate water, kg/s
$\mathrm{TRW}^{\min}$ = minimum amount of total feed water, kg/s
$\mathrm{TRW}^{\max}$ = maximum amount of total feed water, kg/s
T^{amb} = seawater temperature, K
$W^{\min}$ = minimum amount of feed water flow rate in the TMD unit, kg/s
$W^{\max}$ = maximum amount of feed water flow rate in the TMD unit, kg/s
$z_{i,k}^{\mathrm{perm}}$ = mass concentration of permeate
$z^{\max}$ = maximum mass concentration of permeate permissible
z^{seawater} = seawater concentration

Greek letters

β_k^{TMD} = exponent for area cost
δ = membrane thickness, mm
$\xi_{i,k}$ = fraction recovery in TMD unit

Positive variables

$A_{m_{i,k}}$ = membrane area, m^2
AGP = annual gross profit, \$/year
APV = annual permeate value, \$/year
$b_{\mathrm{w}_{i,k}}$ = membrane permeability, kg/(m2 Pa)
$f_{i,i',k}$ = flow rate in the subsequent lines, kg/s
$j_{\mathrm{w}_{i,k}}$ = water flux across the membrane, kg/(m^2 s)
$k_{\mathrm{m}_{i,k}}$ = thermal conductivity of the membrane, kW/(m K)
$p_{\mathrm{wfeed}_{i,k}}^{\mathrm{vap}}$ = water vapor pressure of the feed, Pa
$p_{\mathrm{wperm}_{i,k}}^{\mathrm{vap}}$ = water vapor pressure of the permeate, Pa
$Q_{i,k}^{\mathrm{Heating}}$ = heat consumed in the heater unit, kJ/s
$t_{i,k}^{\mathrm{rej}}$ = temperature of the concentrate, K
$t_{i,k}^{\mathrm{in}}$ = inlet temperature, K
$t_{i,k}^{\mathrm{mix}}$ = outlet temperature of the recycle mixer, K
$t_{\mathrm{m}_{i,k}}$ = average temperature in TMD unit, K
$t_{\mathrm{mfeed}_{i,k}}$ = temperature of the feed at the membrane, K
$t_{\mathrm{mperm}_{i,k}}$ = temperature of the permeate at the membrane, K
$t_{i,k}^{\mathrm{perm}}$ = temperature of the permeate in the bulk, K
$t_{i,k}^{\mathrm{TMD}}$ = feed temperature at bulk in TMD module, K
THeat = total heat consumed, kW
TMA = total membrane area, m^2
TPW = total permeate water, kg/s
TRW = total raw water feed, kg/s
$w_{i,k}^{\mathrm{conc}}$ = flow rate of the concentrate, kg/s
$w_{i,k}^{\mathrm{feed}}$ = flow rate of the raw feed, kg/s
$w_{i,k}^{\mathrm{perm}}$ = flow rate of the permeate, kg/s
$w_{i,k}^{\mathrm{recycle}}$ = flow rate of the recycle, kg/s
$w_{i,k}^{\mathrm{rej}}$ = flow rate of the rejected flow, kg/s
$w_{i,k}^{\mathrm{TMD}}$ = feed flow rate of the TMD unit, kg/s
$x_{\mathrm{wfeed}_{i,k}}$ = molar fraction of water in feed
$x_{\mathrm{NaCl}_{i,k}}$ = molar fraction of NaCl in feed
$z_{i,k}^{\mathrm{rej}}$ = mass concentration in concentrate
$z_{i,k}^{\mathrm{feed}}$ = mass concentration in raw feed
$z_{i,k}^{\mathrm{TMD}}$ = mass fraction in TMD unit feed
Δh_{vw} = latent heat of vaporization for water, kJ/kg
$\gamma_{\mathrm{wfeed}_{i,k}}$ = activity coefficient of the water in feed
$\theta_{i,k}$ = temperature polarization coefficient
$\eta_{i,k}$ = overall thermal efficiency of TMD module

Binary variables

$y_{i,k}$ = binary variables for the existence of TMD units

Free variables

TAC = total annual cost, \$/year

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